Ασκήσεις επιταχυντές με λύσεις – απο κεφ. 3 Tavernier

1. Assume a linear accelerator as shown in Fig. 3.5 and an alternating voltage source of 10 MHz. Assume we want to use it to accelerate electrons. After a few steps, the electrons will have a velocity close to the velocity of light. How long should each of the tubes be to accelerate each electron further?

   Solution:

$$\frac{c}{2f} = 15m$$

2. Assume you have a cyclotron with a magnet of 1.5 Tesla field. The useful diameter of the magnet is 2 m. What is the maximum energy you can reach for protons with this machine?

   Solution: The mass of a proton is $mc^2 = 938$ MeV. From Eq. (3.1), the maximum momentum the proton can have is $Pc = 448.5$ MeV. The maximum energy is given by Eq. (1.5): $E = 102$ MeV.

3. Show that in a cyclotron the distance between the successive orbits becomes smaller as the energy of the particles becomes larger.

   Solution: With each revolution the kinetic energy increases by the same amount. Differentiating Eq. (1.1), we get the relation between the step in energy and the step in momentum

$$c\Delta P\, cP = \Delta E_{\text{kinetic}}(E_{\text{kinetic}} + mc^2)$$
$$c\Delta P = \Delta E_{\text{kinetic}} \frac{(E_{\text{kinetic}} + mc^2)}{cP}$$
$$c\Delta P = \Delta E_{\text{kinetic}} \frac{\sqrt{m^2c^4 + P^2c^2}}{cP}$$
$$c\Delta P = \Delta E_{\text{kinetic}} \sqrt{1 + \frac{m^2c^4}{(cP)^2}}$$
$$\Delta r = \frac{c\Delta P}{cZeB} = \frac{\Delta E_{\text{kinetic}}}{c\,Ze\,B} \sqrt{1 + \frac{m^2c^4}{(c\,Ze\,Br)^2}}$$

   This equation clearly shows that identical steps in energy will correspond to smaller steps in radius as the energy or momentum of the particle increases.

4. Show that the equation for the radius of curvature of the track of a charged particle in a magnetic field: $P = Ze\ B\ r$, can be rewritten as Eq. (3.1).

   Solution: $cP = Ze\ B\ r\ c$. If e is expressed in coulomb, B in tesla and r in meter, we obtain cP in joule. To obtain the quantity Pc in eV, divide the expression by the charge of one electron

$$\{cP\}[eV] = ZB[\text{tesla}]\, r[m]\, c[\text{m/s}] = 2.9979 \times 10^8 ZB[\text{tesla}]\, r[m].$$

5. Assume that to drive a nuclear reactor one needs a beam of protons with an energy of 1 GeV and a beam current of 20 mA. Assume that the accelerator has an efficiency for converting electrical energy to beam energy of 33%. How much electrical power will this accelerator use?

   Solution. If 'e' is the charge of a proton, 'I' the beam current and 'E' the beam energy, the number of protons per second in the beam is given by I[A]/e[coulomb].

   The energy of one proton of E[eV] expressed in joule is $= e$[coulomb] E[eV]

6. What is the speed of a train that has the same kinetic energy as the energy stored in one of the proton beams of the LHC accelerator. A typical train weighs 400 metric tons.

   Solution: There are 3×10^{14} protons in one of the beams of LHC. The total energy E_{tot} in the beam therefore is 33.6×10^7 joule.

   The speed of the train with kinetic energy E_{tot} is given by

$$v[\mathrm{m/s}] = \sqrt{\frac{2E_{\text{tot}}}{m[\mathrm{kg}]}}$$

   This gives 40 m/s or $\approx$150 km/h for the speed of the train.

7. Assume we accelerate protons and make them collide with protons at rest. What should be the energy of the proton beam to produce the same centre of mass energy as is achieved in collisions at the LHC collider.

   Solution: The beam energy of the proton producing the same centre of mass energy in a fixed target collision is given by

$$E_{\text{beam}} = \frac{E_{\text{CM}}^2}{2m_pc^2}$$

   For a centre of mass energy of 14 TeV this gives 2.6×10^{16} eV

8. In the SPS proton synchrotron, the frequency of the RF cavities at the maximum energy of 450 GeV is 200.2 MHz. How much should the frequency be at the injection energy of 10 GeV?

   Solution: The nominal trajectories of the particles should be the same at both energies. We therefore have

$$f = \frac{v}{L} = \frac{c\sqrt{1-(mc^2/E)^2}}{L}$$

   where v and E are the velocity of the protons and L the length of the trajectory. The frequencies at 10 and 450 GeV are hence related by.

$$f_{10} = f_{450}\frac{v_{10}}{v_{450}} = f_{450}\frac{\sqrt{1-\left(mc^2/10\right)^2}}{\sqrt{1-\left(mc^2/450\right)^2}}$$

   The result is $\mathrm{f}_{10} = 199.3$ MHz.

9. Assume a synchrotron for electrons with a beam energy of 1 GeV. What is the power dissipated by synchrotron radiation? Assume that the bending magnets have a field of 2 tesla, that the number of particles stored is 10^{12}, and that 33% of the circumference is occupied by the bending magnets. The rest of the circumference has quadrupoles and straight sections. Neglect the power dissipated in the quadrupoles.

   Solution: Use Eq. (4.2) to find the radius of the beam trajectory in the bending magnets ($r = 1.66$ m). The circumference of the cyclotron is therefore $\approx$30 m and the rotation frequency $\approx 10^7$. The total power dissipated is the power per turn (Eq. 3.3) times the rotation frequency times the number of electrons in the ring. The result is 8×10^4 Watt.