1. Derive the relation between the particle momentum p, the magnetic field B and radius of curvature ρ,

$$p(\mathrm{GeV}/c) = 0.3B(\mathrm{T})\rho(\mathrm{m}).$$

For the particle to stay in orbit, the centripetal acceleration must equal the force exerted by the magnetic field,

$$\frac{(\gamma m)v^2}{\rho} = qvB.$$

Since $p = \gamma mv$, and $q = e$ (usually particles to be accelerated have charge $\pm e$),

$$p = eB\rho.$$

At this point, the units are

$$\begin{aligned}
[p] &= \frac{\mathrm{kg\,m}}{\mathrm{s}} \\
[B] &= \mathrm{T} = \frac{\mathrm{kg}}{\mathrm{C\,s}} \\
[\rho] &= \mathrm{m} \\
[q] &= \mathrm{C} \\
[eB\rho] &= \frac{\mathrm{C\,kg\,m}}{\mathrm{C\,s}} = \frac{\mathrm{kg\,m}}{\mathrm{s}}
\end{aligned}$$

So, we need to calculate the conversion from $\frac{\mathrm{kg\,m}}{\mathrm{s}}$ to GeV/c:

$$\begin{aligned}
1\,\mathrm{eV/c^2} &= 1.78 \cdot 10^{-36}\,\mathrm{kg} \\
1\,\mathrm{GeV/c} &= 10^9 \cdot 3 \cdot 10^8 \mathrm{m/s} \cdot 1.78 \cdot 10^{-36}\,\mathrm{kg} = 5.34 \cdot 10^{-19} \frac{\mathrm{kg\,m}}{\mathrm{s}}
\end{aligned}$$

and include

$$e = 1.602 \cdot 10^{-19}\,\mathrm{C}$$

giving

$$p[\mathrm{GeV/c}] = \frac{1.602 \cdot 10^{-19}}{5.34 \cdot 10^{-19}} \mathrm{B[T]}\rho[\mathrm{m}] = 0.3\mathrm{B[T]}\rho[\mathrm{m}]$$

2. Knowing that dE/dx is function of velocity only, show that the ratio of the ranges in a material of charged particles of the same initial velocity equals the ratio of their masses.

$$\frac{dE}{dx} = f(v) \Leftrightarrow dE = f(v)dx$$

While the particle penetrates into the material, its energy decreases from E_0 to m in distance l.

$$\int_{E_0}^{m} \frac{dE}{f(v)} = \int_0^l dx = l.$$

The energy of the particle can be expressed as $E(v) = \gamma(v)m$, and so the integration can be switched to be done in terms of the particle velocity v,

$$l = \int_{E_0}^{m} \frac{d(\gamma m)}{f(v)} = \int_{v_0}^{0} m\frac{\partial\gamma}{\partial v}dv\frac{1}{f(v)} = m\int_{v_0}^{0} dv\frac{1}{f(v)}\frac{\partial\gamma}{\partial v}$$

We are interested in the ratio of ranges of two particles with the same initial velocity and different masses, m_1 and m_2,

$$\frac{l_1}{l_2} = \frac{m_1\int_{v_0}^{0} dv\frac{1}{f(v)}\frac{\partial\gamma}{\partial v}}{m_2\int_{v_0}^{0} dv\frac{1}{f(v)}\frac{\partial\gamma}{\partial v}} = \frac{m_1}{m_2}$$

3. dE/dx is used for particle identification. Using the simplified form of Bethe-Bloch formula,

$$\frac{dE}{dx} \propto \frac{1}{\beta^2}\ln\gamma,$$

calculate the ratio of energy loss for $p = 5$ GeV/c pions and kaons.

This is rather straightforward.

$$R = \frac{\frac{dE}{dx}|_\pi}{\frac{dE}{dx}|_K} = \frac{\frac{1}{\beta_\pi^2}\ln\gamma_\pi}{\frac{1}{\beta_K^2}\ln\gamma_K} = \frac{\beta_K^2\ln\gamma_\pi}{\beta_\pi^2\ln\gamma_K}.$$

$$\beta = \frac{p}{E} = \frac{p}{\sqrt{m^2+p^2}}$$

and

$$\gamma = \frac{E}{m} = \frac{\sqrt{p^2+m^2}}{m}$$

and thus

$$R = \frac{\frac{p^2}{m_K^2+p^2}\ln\frac{\sqrt{p^2+m_\pi^2}}{m_\pi}}{\frac{p^2}{m_\pi^2+p^2}\ln\frac{\sqrt{p^2+m_K^2}}{m_K}} = \frac{(m_\pi^2+p^2)\ln\frac{\sqrt{p^2+m_\pi^2}}{m_\pi}}{(m_K^2+p^2)\ln\frac{\sqrt{p^2+m_K^2}}{m_K}}$$

Plugging in the numbers, $m_\pi = 0.140$ GeV/c^2, $m_K = 0.490$ GeV/c^2 and $p = 5$ GeV/c,

$$R = \frac{(25.0196)\ln\frac{\sqrt{25.0196}}{0.140}}{(25.2401)\ln\frac{\sqrt{25.2401}}{0.490}} = \frac{89.47}{58.75} = 1.52$$

4. a) Two particles of masses m_1 and m_2, both with momenta p, travel between two scintillation counters, which are a distance L apart. Show that the difference in their flight times decreases like p^{-2} for large momenta.
b) Calculate the minimum flight path necessary to distinguish pions from kaons if they have a momentum of 3 GeV$/c$ and the time-of-flight can be measured with an accuracy of 200 ps.

a) The velocity can be expressed in terms of particle mass and momentum as

$$v = \beta c = \frac{p}{E} c = \frac{pc}{\sqrt{p^2 + m^2}}.$$

The time it takes a particle to traverse distnace L is

$$t = \frac{L}{v},$$

and so

$$\Delta t = L(\frac{1}{v_1} - \frac{1}{v_2}) = \frac{L}{pc}(\sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2}) = \frac{Lp}{pc}(\sqrt{1 + \frac{m_1^2}{p^2}} - \sqrt{1 + \frac{m_2^2}{p^2}})$$

Since we are looking at large momentum case, $p \gg m$, and we can expand $\sqrt{1 + \frac{m^2}{p^2}}$ as a power series,

$$\sqrt{1 + \frac{m^2}{p^2}} = 1 + \frac{m^2}{2p^2} + \mathcal{O}(\frac{1}{p^4}).$$

Therefore

$$\Delta t = \frac{L}{c}((1 + \frac{m_1^2}{2p^2}) - (1 + \frac{m_2^2}{2p^2})) = \frac{L}{2c}\frac{m_1^2 - m_2^2}{p^2} \propto p^{-2}$$

b) Solving from a), $(p \gg m_\pi, m_K)$

$$L = \frac{2c\,\Delta t p^2}{m_1^2 - m_2^2} = \frac{2 \cdot 3 \cdot 10^8\,\mathrm{m/s} \cdot 200 \cdot 10^{-12}\,\mathrm{s} \cdot 9\,\mathrm{GeV}^2}{(0.49^2 - 0.14^2)\,\mathrm{GeV}^2} = 4.9\,\mathrm{m}$$

5. What is the threshold momentum for Cherenkov radiation for pions, kaons and protons in
a) $PbWO_4$ (n=2.20)
b) C_5F_{12} gas (n=1.0018)?

The angle θ of Cherenkov radiation is

$$\cos\theta = \frac{1}{n\beta},$$

where n is the refractive index of the medium. The limiting case is when the opening angle $\theta = 0$, or $\cos\theta = 1$, in which case

$$\beta = \frac{1}{n}.$$

On the other hand,

$$p = \gamma m v = \frac{m\beta}{\sqrt{1-\beta^2}} = \frac{m}{n\sqrt{1-\frac{1}{n^2}}} = \frac{m}{\sqrt{n^2-1}}.$$

Incorporating the numbers,

		π	K	p
	mass	0.140	0.490	0.940
$PbWO_4$	n=2.20	0.071	0.250	0.480
C_5F_{12}	n=1.0018	2.332	8.163	15.660